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THE FRACTURE MECHANICS APPROACH TO FATIGUE

by

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ABSTRACT

Since a considerable portion of fatigue life is associated with the crack growth period, a study of crack propagation seems appropriate. Justification is found for attempting treatment of crack growth as a continuous process and at the continuum level. An analysis of growth rates making use of the general approach and parameters of fracture mechanics is presented. The use of stress-intensity-factors enables direct comparisons of crack growth rates between different configurations. The 4th power law of crack growth is discussed and an accumulation of damage model which it suggests is evaluated with some preliminary data from random loading tests.

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THE FRACTURE MECHANICS APPROACH TO FATIGUE

by Paul C. Paris[1]

INTRODUCTION

Fatigue has been approached in many ways. Recently, the study of crack growth has received wider attention. It has been observed that cracks form very early in the fatigue life of materials. Therefore, a study of rates of crack growth may lead to a better understanding of the overall problem of prediction of fatigue life.

The introduction of a crack in a stressed body leads to a redistribution of stresses near the crack, especially in the region adjacent to the crack tip. Since the crack growth process takes place in that region, a stress analysis of the body including the crack will be regarded as mandatory in the discussion. However, the stress analysis will be confined to an elastic analysis of the redistribution of stresses and the overall process will be viewed from a macroscopic level. Moreover, the growth process will be considered to be continuous in order to attempt

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a synthesis of crack growth from the simplest possible view point. The advantages and/or disadvantages of such an elementary treatment will be assessed.

Upon viewing crack growth as described, i.e. at the continuum level, two main sub-problems arise:

(1) The influence of configuration of a body including the crack on crack growth rates for a given type of load-time history.

(2) The influence of various types of load-time histories on crack growth rates.

These sub-problems will be treated separately. The results of an elastic stress analysis will be shown to resolve the influence of configuration. Subsequently, exploring the influence of load-time histories requires formulation of hypotheses of "Accumulation of Damage" in the vicinity of a crack tip. A very simple damage law will be compared with random load test results.

STRESS ANALYSIS OF CRACKED BODIES

In general, fatigue cracks tend to grow perpendicular to principle tensile stress directions in bodies. In the majority of cases, this means that the crack plane is a plane of (at least local) symmetry of both the configuration and stress field. Therefore this discussion will be restricted to symmetrical cases, though generalization to quite arbitrary situations is not beyond the scope of the techniques to be employed.

In recent years, two methods of elastic stress analysis of cracked bodies have been developed [1,2]. These methods have been used for treating their strength and have also been shown to apply to fatigue crack growth [3,4]. Both of these applications correlate crack growth rates on the basis of local, crack tip, stress parameters and may be shown to be effectively equivalent for that purpose [5]. As a matter of arbitrary choice, Irwin's crack tip stress-intensity-factor [1] will be employed as the parameter in this discussion.

The elastic stress field surrounding the tip of a crack, where the crack plane is a plane of symmetry, is given by [1]:

$$\sigma_x = \frac{K}{\sqrt{2r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K}{\sqrt{2r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\tau_{xy} = \frac{K}{\sqrt{2r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{1}$$

where r and θ are cylindrical position coordinates measured from the leading edge of the crack and the plane of extension of the crack, respectively. The stress-intensity-factor, K, appears linearly in these equations of elastic stress. Consequently, K must depend linearly on the intensity of the applied load, P, and must reflect the influence of the configuration including the crack size, a, or

$$K = P\ f(a) \tag{2}$$

The specific functional form in Eq. (2) depends on the elastic solution of the boundary value problem for the particular configuration which may be of interest. Many solutions are available for both two and three dimensional problems, e.g. [1,6] and many other sources. They shall be employed herein without further discussion.

Though strictly speaking during fatigue crack growth the crack length is a function of time, during the interval from one peak in the load to the next, it changes very little. As a consequence, the wave form of the local stress field surrounding the crack tip as reflected by the time history of the stress-intensity-factor, K(t), depends significantly only on the wave form of the time history of the load, P(t). That is to say that from Eq. (2):

$$K(t) = P(t).f(a) \tag{3}$$

Where f(a) is a very slowly changing amplifying factor. Consequently, the wave form of K(t) is regarded as identical to P(t) for the purpose of this discussion. This assumption is examined in the section to follow.

SINUSOIDAL LOADING

On the basis of the preceding arguments, bodies subjected to sinusoidal loading experience effectively sinusoidal variations of the fields of stress surrounding their crack tips as measured by K(t). Therefore, the rate of fatigue crack growth in a given material should depend upon the range of variation of the stress-intensity-factor, ΔK, and its mean, K_{mean}. For convenience, it is better to use the relative mean, γ, which by incorporating Eq. (3) leads to

$$\gamma = \frac{K_{mean}}{\Delta K} = \frac{P_{mean}}{\Delta P} \tag{4}$$

Figure 1 shows crack growth rate test data for 7075-T6 aluminum alloy sheet specimens correlated on the basis of the range of the stress-intensity-factor, ΔK, for a given relative mean, γ. The data shown are taken from tests using two quite different configurations. The first is of the usual type, a panel subjected to uniform stress, σ, with a central crack of length, 2a. The stress-intensity-factor* for this configuration [1] is (in the form of Eq. (2)):

$$K = \sigma\sqrt{a} \tag{5}$$

The second configuration is a centrally crack panel with equal and opposite concentrated forces, F, applied to the upper and lower crack surface at the center of the crack, i.e. so called "wedge forces". Its stress-intensity-factor* is [1]:

$$K = \frac{F}{\pi\sqrt{a}} \tag{6}$$

*Correction factors for the finite width of panels have been omitted for clarity.

where F denotes force per unit thickness of the sheet.

The correlation of data on the basis illustrated in Figure 1 has been discussed in detail earlier [3,4, 5,7]. Of particular interest here is the fact that data from quite different configurations are shown to concur by means of a local crack tip stress parameter, K. It should be pointed out that the N.A.S.A. Method [4] using a similar local stress parameter is also capable of exacting such concurrences.

Moreover, special emphasis should be placed on the fact that the data from the two configurations chosen for Figure 1 permit a critical examination of the effect of the assumptions of Eq. (3). From Eqs. (5) and (6), it can be observed that, as the crack length, a, increases, the change in f(a) is opposite in sign. Nevertheless, the data on Figure 1 are apparently unaffected, hence the assumption of quasi-stationary crack length is shown to be justified under quite extreme conditions. A more thorough examination [8] of existing data to assess this assumption nets the same result.

RANDOM LOADING

It is pertinent to turn attention to stochastic loading, i.e., stationary random processes of Gaussian distribution. Though such loadings are of practical interest for structures encountering atmospheric turbulence or waves at sea, for the time being they shall be treated as academic, in furtherance of a basic understanding of the nature of fatigue crack growth.

The sequence of peaks in a stochastic loading is to say the least devious. The time record of a typical random load shows this character; see Figure 2. Several sources report [9,10,11] that delays in the crack growth process occur following overloads applied during a sinusoidal loading test. The delays are a complete cessation of growth for from hundreds to millions of cycles. Therefore, in examining crack growth under random loadings, one might expect delays to follow high peaks in load. This does not appear to be the case. Figures 3 and 4 show electron-micrographs (2500x and 3000x) of a crack surface generated under the loading shown in Figure 2. A count of the markings reveals that a new ring is formed for each peak in the loading. A similar observation was made by Fuller [12]. Therefore, it appears that crack growth under random loading may be regarded as a reasonably continuous process.

It follows then that it is feasible to attempt to use the data correlation technique employed on Figure 1 for individual random loadings. In order to give the broadest definition to an "individual loading" a statement of the conditions for equivalence must be made.

A stochastic process is completely defined by its power spectrum. Then, assuming that the crack growth process is insensitive to rate of loading (i.e. frequency), the statement of equivalence of wave form is:

> All random loadings whose power spectra may be made identical by scaling their coordinate axes with constant factors are equivalent in wave form.

The proof that this statement of equivalence leads to load-time histories that differ only by a factor in magnitude and/or a factor in rate of application is within the scope of the usual mathematical analysis of random processes [8].

Again, applying Eq. 3, since the intensity of the field of local stress, K, is proportion to the load, P, in the quasi-stationary sense, the power spectrum of the stress intensity factor, $S_K(\omega)$ is related to the power spectrum of the load, $S_p(\omega)$ by*

$$S_K(\omega) = S_p(\omega) \cdot [f(a)]^2 \tag{7}$$

Therefore equivalent load power spectra, as defined above, will lead to equivalent wave forms of stress-intensity-factors. If, in addition, the magnitudes of variation of the stress-intensity-factors are respectively the same for cracks in two different specimens of a given material, the average crack extension rates (per peak in load) would be expected to be similar. In this manner, crack growth rate data from 7075-T6 aluminum alloy tests under equivalent load power spectra of the type shown in Figure 5 are compared on Figure 6. In each test, a mean load of proportionate intensity has been applied in order to avoid compression. The magnitude of variation of the stress-intensity-factor, as measured by its average

*Eq. (7) follows from Eq. (3) and the definition of power spectra given in Eqs. (12) and (13) through to peak rise, $\overline{h_k}$, is plotted as the ordinate. The average change in crack length, 2a, per peak (or rise), $\overline{d(2a)/dN}$ is plotted as the absissa. The observed correlation of data into a single curve is regarded as justification of the methods employed.

In preparing test data for Figure 6, $\overline{h_k}$ and $\overline{N}$ have been evaluated from an actual count of the rises in the load-time history and Eq. (5) has been used to appropriately convert the data into terms of the stress-intensity-factor. However, both could have been computed directly from the load power spectrum. Using Eq. (3),

$$\overline{h_K} = \overline{h_p}\, f(a) \tag{8}$$

The average rise in load, $\overline{h_p}$, is given by [8,]

$$\overline{h_p} = \frac{M_2}{\sqrt{M_4}} \tag{9}$$

and the average number of peaks per unit time $\overline{N}$ is [8,12,13]

$$\overline{N} = \frac{1}{2\pi}\sqrt{\frac{M_2}{M_4}} \tag{10}$$

where M_r are moments of the load power spectrum,

$$M_r = \int_{-\infty}^{\infty} \omega^r \, S_p(\omega) \, d\omega \tag{11}$$

and since various authors differ on definition of power spectrum, it is defined herein as

$$S_p(\omega) = \lim_{T \to \infty} \frac{1}{2T} \left| F^T(\omega) \right|^2 \tag{12}$$

where

$$F^T(\omega) = \int_{-T}^{T} P(t) \, e^{-i\omega t} \, dt \tag{13}$$

Though Eqs. (8) through (13) may be useful in reducing crack growth data, they are not essential to this discussion. They merely indicate that computation of quantities involved in the analysis represented by Figure 6 is possible.

Thus far, it has been shown that stress-intensity-factors permit direct comparisons of crack growth rates in bodies of different configuration but subjected to loadings with equivalent wave form. It remains to attempt means of comparing propagation rates under one type of loading with another. As a means to this end a digression into power laws of fatigue crack propagation is appropriate.

CRACK PROPAGATION LAWS

A variety of crack propagation laws have been proposed to date [5 and references cited therein]. All of these laws attempt to relate crack growth rates under sinusoidal loading to the applied stress levels, the crack length and material parameters. However, data representations such as Figures 1 and 6 serve this purpose. Therefore the main motivation for subsequently stating a propagation law is to provide insight into the parameters useful in analyzing "accumulation of damage". Consequently, a simple law of wide generality is sought.

The most appealing type of law for this purpose is the power law, i.e. for sinusoidal loading,

$$\frac{d(2a)}{dN} = C \cdot (\Delta K)^m \tag{14}$$

where C depends upon the relative mean load, γ, and the material (and frequency if its effect is significant). Several investigations of this type of law have been reported recently [5,8,13,14]. The high dependence of crack growth rate on range of stress, i.e. ΔK, compared to the relative mean, γ, allows consideration of this type law as representative of the main trend of crack growth behavior.

The exponent in Eq. (14) can be most readily evaluated from the slope of a log-log plot of test data. The widest range of data available has been observed for 7075-T6 aluminum alloy at $\gamma \cong 0.5$; it is shown on Figure 7. The straight line shown on this figure corresponds to $n = 4$. Figures 8 and 9 show the concurrence with $n = 4$ of more limited data on other materials. The apparent agreement with the same exponent for a fairly wide variety of materials has interesting implications.

If the general behavior of fatigue crack propagation is highly dependent on the type of micro mechanisms of crack growth taking place in the crack tip region, then it would be expected that different microstructures would lead to rather different laws of growth. Therefore, the behavior (of body-center-cubic and face-center-cubic materials with various alloying constituents) according to the same law is evidence that phenomena at the continuum level are most important in establishing general trends. Laird and Smith [15] report supporting conclusions based on microscopic observation of the mechanism of crack propagation. Hopefully, this means that it is feasible to attempt to formulate "accumulation of damage laws" and the like entirely in terms of continuum variables.

ACCUMULATION OF DAMAGE IN CRACK GROWTH

Crack growth itself may be regarded as damage, i.e. an accumulation of length until final failure occurs. The rates of propagation for various load intensities in sinusoidal block loading tests may be predicted from curves such as Figure 1. Subsequently, these rates may be numerically integrated, including delays, to estimate crack propagation life. Though these estimates are reasonable for block loading [7] (and in fact predict a proper difference in life under two step load programming when the order is reversed), they are of little help in predicting, for example, the rate of crack growth under random loading from sinusoidal loading test data. Consequently, it is desirable to view damage in a more detailed way than merely the extension of the crack itself. The desired result is a damage law capable of handling arbitrary wave-forms of loading.

Thus far the elastic stress field surrounding the crack tip and extension of crack itself have been used as a limit of elaboration in the approach. Actually, the material just ahead of the crack tip sustains high stresses, see Eq. (1) and as a consequence yielding occurs. Since plasticity is irrecoverable deformation or "damage", recognition of the plastic enclave accompanying the crack tip is the next natural step in further elaboration.

Irwin [15] has developed a means of estimating the size of the plastic zone at a crack tip in a body subjected to a single monotonic loading. Presuming the elastic stress equations, Eqs. (1), are approximately correct up to the boundary of the plastic zone, the location of the boundary can be estimated. In particular the width, w, of the enclave directly ahead of the crack, $\theta = 0$, is the radius at which the stresses, Eqs. (1), violate the yield condition. As a result:

$$w = \frac{K^2}{2\sigma_{y.P.}^2} \tag{15}$$

However, for a load time history which fluctuates thru peaks and troughs, as with fatigue, this estimate must be revamped. Each time the load takes an excursion, a rise or a fall, the infinite theoretical stress concentration factor at a crack tip implies that a new plastic zone forms in tension or compression respectively, superimposed over the previously formed zones. Elements of material within these zones experience alternating plasticity. Schematically this concept is illustrated on Figure 10. At point 1, the last plastic zone formed was in tension (marked +). Due to the drop to 2 in the stress-intensity-factor, h_k, the stresses attempt a negative excursion. The excursion is estimated by inserting, h_k, for k in Eqs. (1), except where it exceeds 2 $\sigma_{y.P.}$, since compressive yielding will occur (marked -). Hence, the size of the new (compression) zone of plasticity, w_h, caused by the excursion can be estimated by replacing k by h_k and $\sigma_{y.P.}$ by $2\,\sigma_{y.P}$ in Eq. (15). The result is:

$$w_h = \frac{h_k^2}{8\,\sigma_{y.P.}^2} \tag{16}$$

It may be used to estimate the successive yield zone sizes due to a succession of rises and falls.

Of utmost importance is the fact that in this result the peak to peak rises and falls in the K-time history are shown to be of major significance. They are noted to be the cause of creation of new zones of plasticity, i.e. additional damage. It is also appropriate to observe that the relative mean load is of no consequence in Eq. (16), though in more refined estimates of plastic zone size it does show a minor role, e.g. [17,18].

It is informative to compare these results to Eq. (14) with n = 4 as observed from figures 7, 8, and 9. Under sinusoidal loading the crack growth rate is proportional to the 4th power of the range, ΔK, or rises and falls, h_k. Since h_k has been shown to be the important variable in generating damage, it is tempting to generalize Eq. (14) by substituting it for K, whereby:

$$\overline{\frac{d(2a)}{dN}} = C \; \overline{h_K^4} \tag{17}$$

where $\overline{d(2a)/dN}$ is interpreted as the average extension of the crack per rise and $\overline{h_K^4}$ is the average of the 4th power of the rises in the quasi-stationary K-time history.

THE WORK RATE MODEL OF CRACK GROWTH

The analogue of Eq. (17) has been derived on several bases [8,14,17,18,19], but usually the analysis is restricted to sinusoidal loading. It is sufficient here to rederive it in general on a basis most common to fracture mechanics. The Griffith-Irwin fracture model arose as the result of the assumption that the work absorbed per unit increase in surface area of a crack is constant. Adapting this same assumption to fatigue crack growth results in:

$$\overline{\frac{d(2a)}{dN}} = \overline{\frac{dW}{dN}} \cdot C_1 \tag{18}$$

where $\overline{\frac{dW}{dN}}$ is defined as the average plastic work per rise in the load time history.

The work done in an excursion of the load will be proportioned to the volume of the plastic zone per unit length of the crack front, i.e. proportional to the square of the plastic zone size. Then from Eq. (16):

$$W = C_2\, w_h^2 = \frac{C_2\, h_K^4}{64\, \sigma_{y.p}^4} \tag{19}$$

substituting Eq. (19) into Eq. (18) and lumping the constants,

$$\overline{\frac{d(2a)}{dN}} = C\, \overline{h_K^4} \tag{20}$$

This result is identical to Eq. (17) and as an interpretation of it suggests a means of evaluating it, i.e. an evaluation of the constancy of hysteresis energy associated with fatigue crack growth is appropriate. Unfortunately, no energy absorption test data are available. The results of some existing random load, however, may be used for an assessment of the "accumulation of damage law" represented by Eq. (20).

THE COMPARISON OF GROWTH RATES UNDER SEVERAL RANDOM LOADINGS

In the earlier section on random loading, adoption of the continuous process concept and Eq. (3) with its assumption of quasi-stationary crack length was justified. Therefore, a rise or fall in the stress-intensity-factor, h_k , is the direct result of a rise or fall in the load, h_p. That is to say that:

$$h_K = h_p \; f(a) \tag{21}$$

where f(a) is a known function for the configuration or interest, as in Eq. (5) or (6). In order to compute the average of the 4th powers of the rises or falls, $\overline{h_K^4}$, it is desirable to know the distribution (probability density) function, $q(h_K)$, since:

$$\overline{h_K^4} = \int_0^\infty h_K^4 \; q(h_K) \, d h_K \tag{22}$$

However, it is usually more convenient to compute quantities in terms of the loading itself, or incorporating Eq. (21) into Eq. (22)

$$\overline{h_K^4} = f(a)^4 \int_0^\infty h_p^4 \, q(h_p) \, d(h_p) \tag{23}$$

Though the distribution of rises and falls in load, $q(h_p)$, can theoretically be computed from the power spectrum [20], such a computation is very difficult. Approximate means are being developed to ease the computation [21,22]. In the meantime, the distribution function can be evaluated numerical from the load time history, as will be the case in the data to follow.

Information of the above nature is available on the five random loadings whose normalized power spectra are shown in Figure 11. Three of these, A,B, and C, were employed in ordinary fatigue studies by Fuller [12] and another, E, by Leybold [23]. Samples of the time histories of these loadings are shown in Figure 12. From Figures 11 and 12, it can be noticed that the broader the band width (of frequencies) in the power spectrum the more devious the load-time history appears.

The normalized probability density functions of rise and fall, $q(h_p)$, for each of the loadings appear in Figure 13. The distribution function for loading, A, the narrow band spectrum is close to its theoretical values, i.e. a Rayleigh distribution [24]. Though these curves exhibit only a mild shift to the left with increasing bandwith, it must be remembered that in accordance with the integral in Eq. (23), the fourth moment of each about the vertical coordinate axis is of interest. These moments for each are:

A=128, B=88.4, C=101.1, D=64.0 and E=40.4.

Crack growth rate data in 7075T6 aluminum alloy for three of these loadings, A,B and C, are shown in Figure 14. Appropriate mean loads were applied during the tests to prevent compression. Analogous to figures 7 through 9 and Eq. (14), Eq. (20) dictates that log $\overline{d(2a)/dN}$ vs. log $\sqrt[4]{\overline{h_K^4}}$ be shown for comparisons in terms of most nearly equivalent parameters. The data on Figure 14 group within a scatter band of a factor of 1.2 in growth rates. A scatter of this order is usual in reproductibility of tests under sinusoidal loading, hence it is regarded as acceptable without interpretation.

For the random load analysis a new definition of relative mean load, γ, is required. Generalizing the previous definition, Eq. (4), so that according to Eq. (20) C may be regarded as a particular function of γ for a given material under arbitrary type of loading, results in

$$\gamma = \frac{K_{mean}}{\sqrt[4]{\overline{h_K^4}}} = \frac{P_{mean}}{\sqrt[4]{\overline{h_P^4}}} \tag{24}$$

In Figure 15 the data from Figure 14 are replotted in addition to data on 7075-T6 aluminum alloy under sinusoidal loading with a similar, γ. Though the results of this plot do not induce as much optimism as Figure 14, it shows that, so far as can be examined by data available to date, this model appears to be a farily reasonable representation. However, more data from different types of loadings and over wider ranges of crack growth rates must be obtained before any final conclusions can be drawn.

Moreover, a shortcoming of the model may already be anticipated. It "counts" the effects of rises and falls in load-time histories with the same weight regardless of the sequence in which they occur. Even though sequence effects may be minor compared to the effects of the rises and falls and the mean, there are surely some effects of sequence. A more elaborate model will be required to account for them. Some analogous studies on "counting" methods have already begun on a fatigue life basis [23,25].

CONCLUSIONS

1. The process of crack growth may be regarded as continuous and the crack length may be regarded as quasi-stationary in analyzing crack growth rates.

2. The elastic stress analysis of cracked bodies and the resulting "fracture mechanics" parameter, the crack tip stress-intensity-factor, is useful in analysis of fatigue crack growth rates.

3. Application of stress-intensity-factors and equivalence of their time histories leads to methods for correlation of crack growth rates in bodies of a given material of arbitrary configuration under individual types of loading.

4. All data on crack growth rates under sinusoidal load for a variety of different materials, generally agree in their broad trend with a law of the type

$$\frac{d(2a)}{dN} = C\,(\Delta K)^4$$

5. Models of "accumulation of damage" appear to be both feasible and reasonable upon elaboration of the gross nature of plasticity in the zone adjacent to a crack tip.

6. A simple work rate model is presented which shows promising results through some preliminary correlations of growth rate data under various random loadings. However, a shortcoming is acknowledged and more data is required to permit a thorough evaluation.

ACKNOWLEDGEMENT

The data on crack growth rates under random loading and the analysis of the corresponding load-time histories was accomplished at the Boeing Company, Airplane Division, Renton, Washington through the efforts of Mr. Samuel H. Smith with the aid of Mr. Ronald G. White. Early publication of their complete data is anticipated. The aid of Mr. Edward Fulmer in preparing the data for this paper is also gratefully acknowledged.

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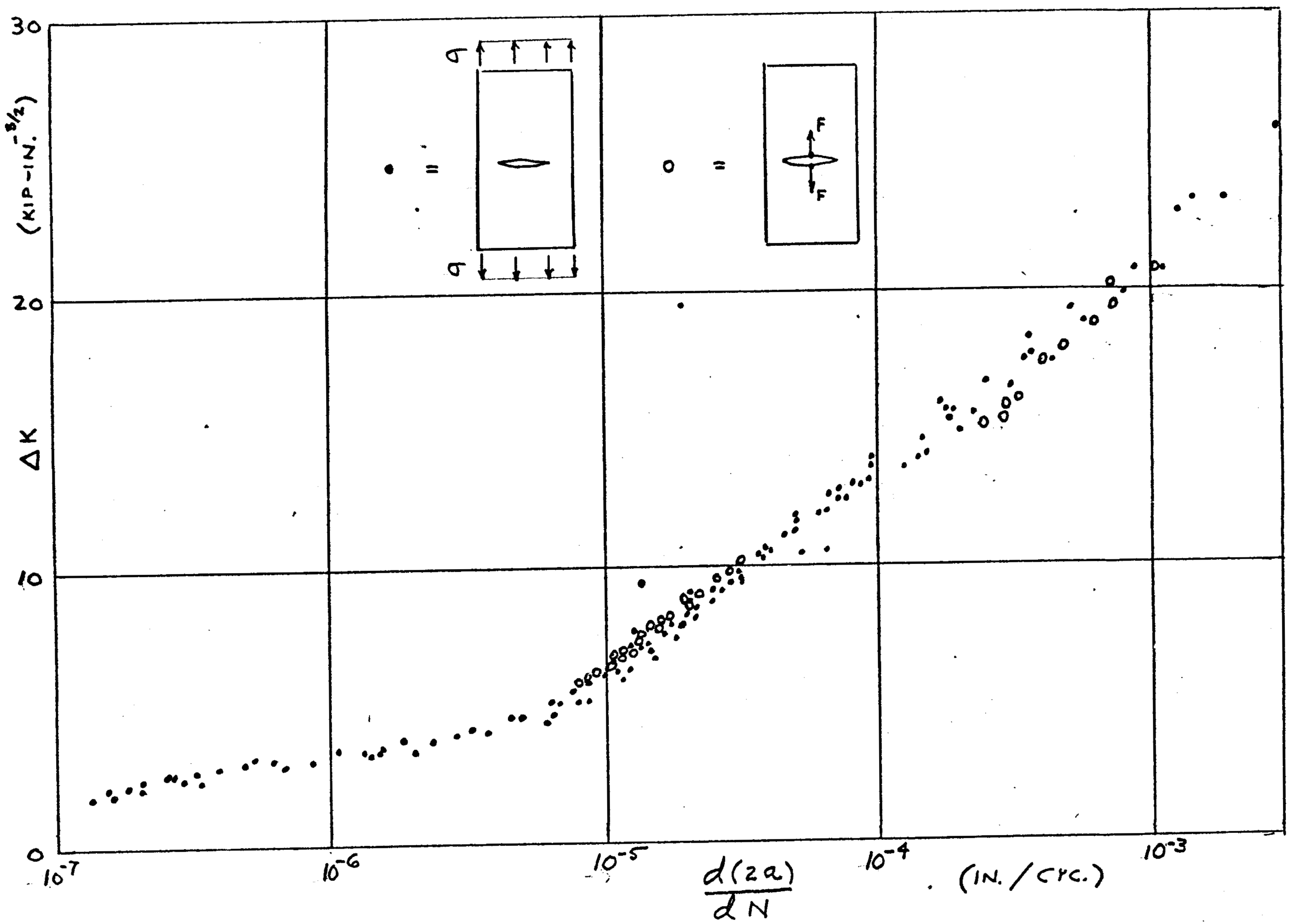

FIGURE 1 — CORRELATION OF 7075-T6 DATA

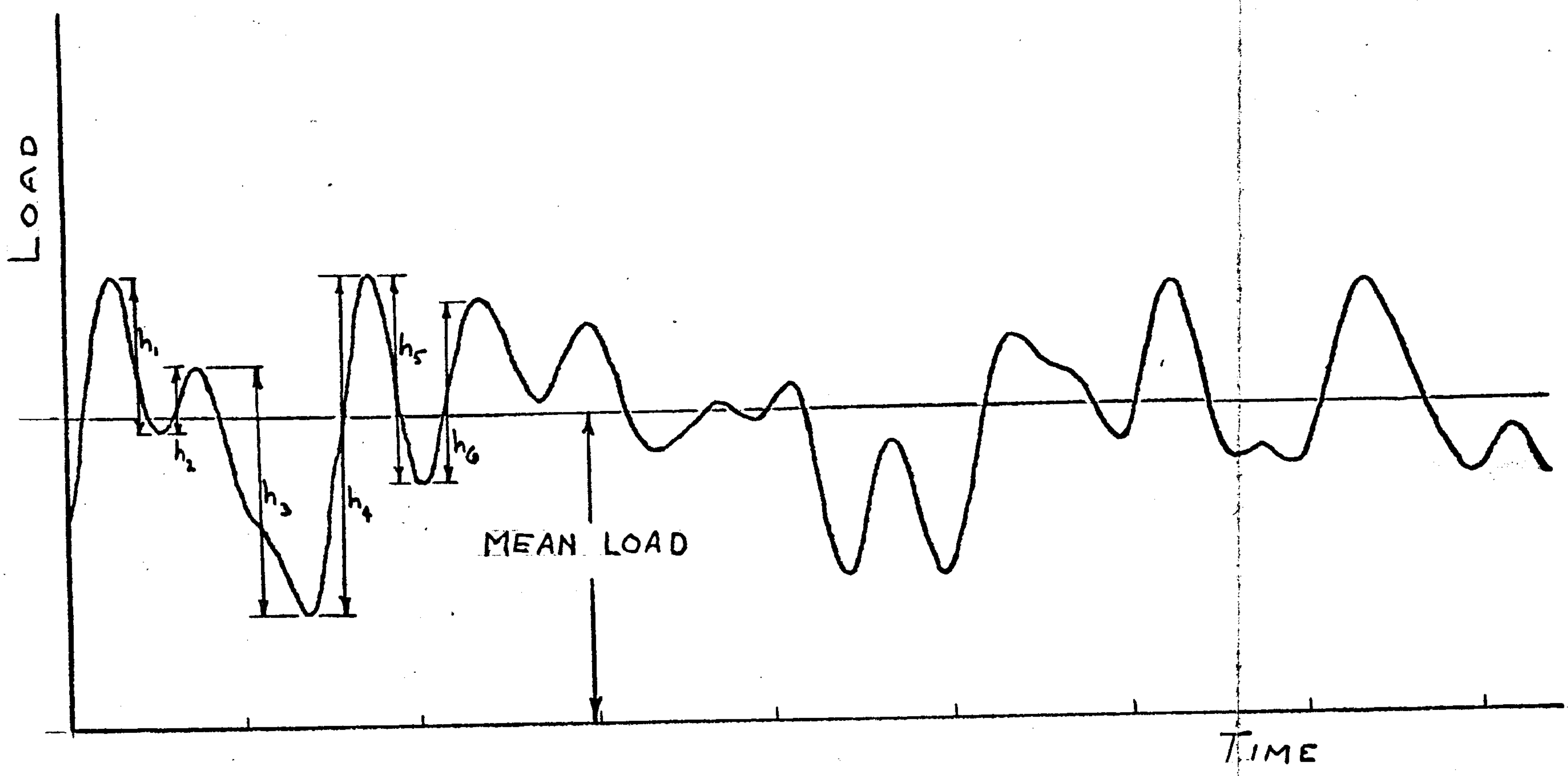

FIGURE 2 — TYPICAL RANDOM LOAD (ACTUAL RECORD)

FIGURE 3 - CRACK SURFACE FROM RANDOM LOAD TEST (2500x)

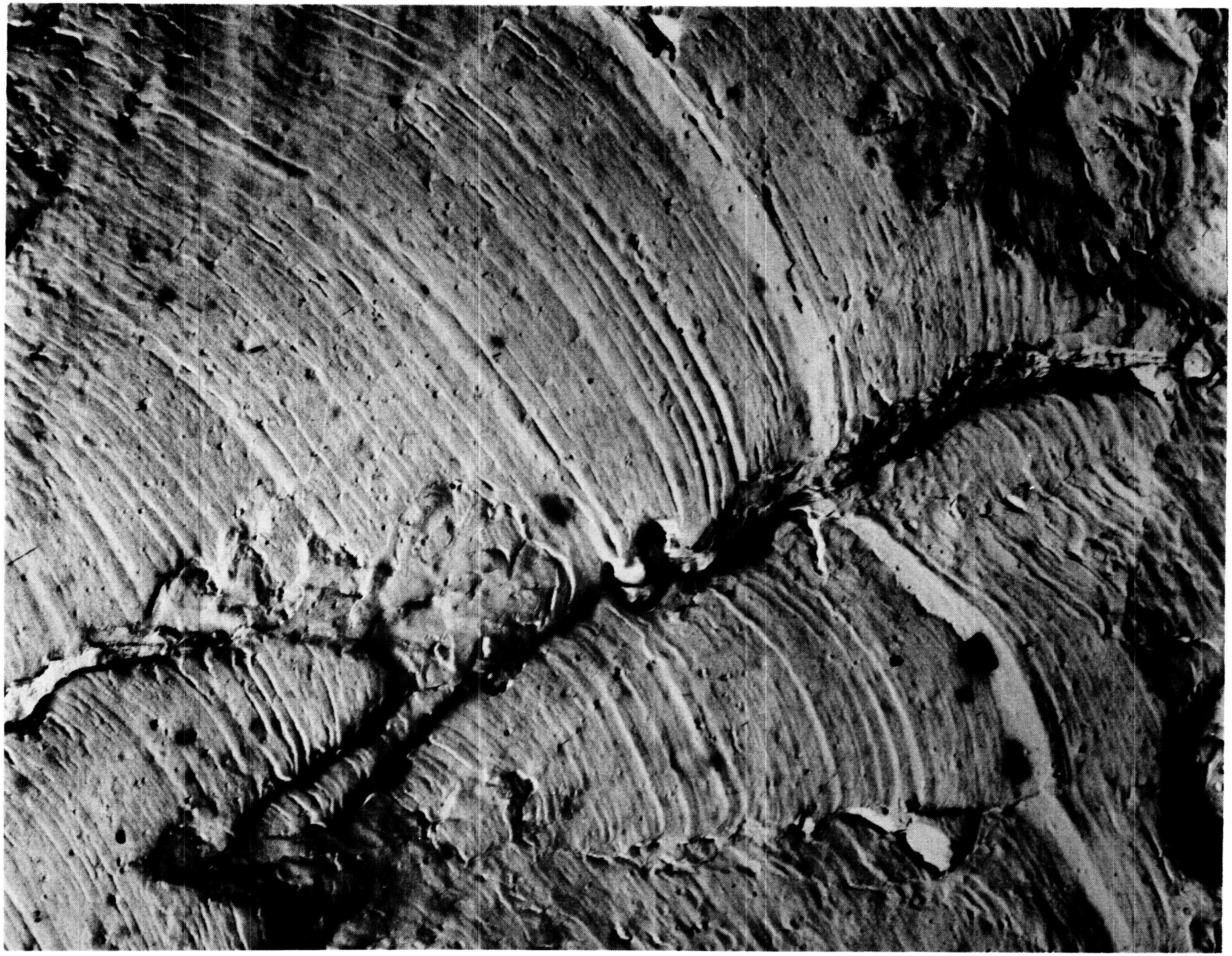

FIGURE 4 - CRACK SURFACE FROM RANDOM LOAD TEST (3000x)

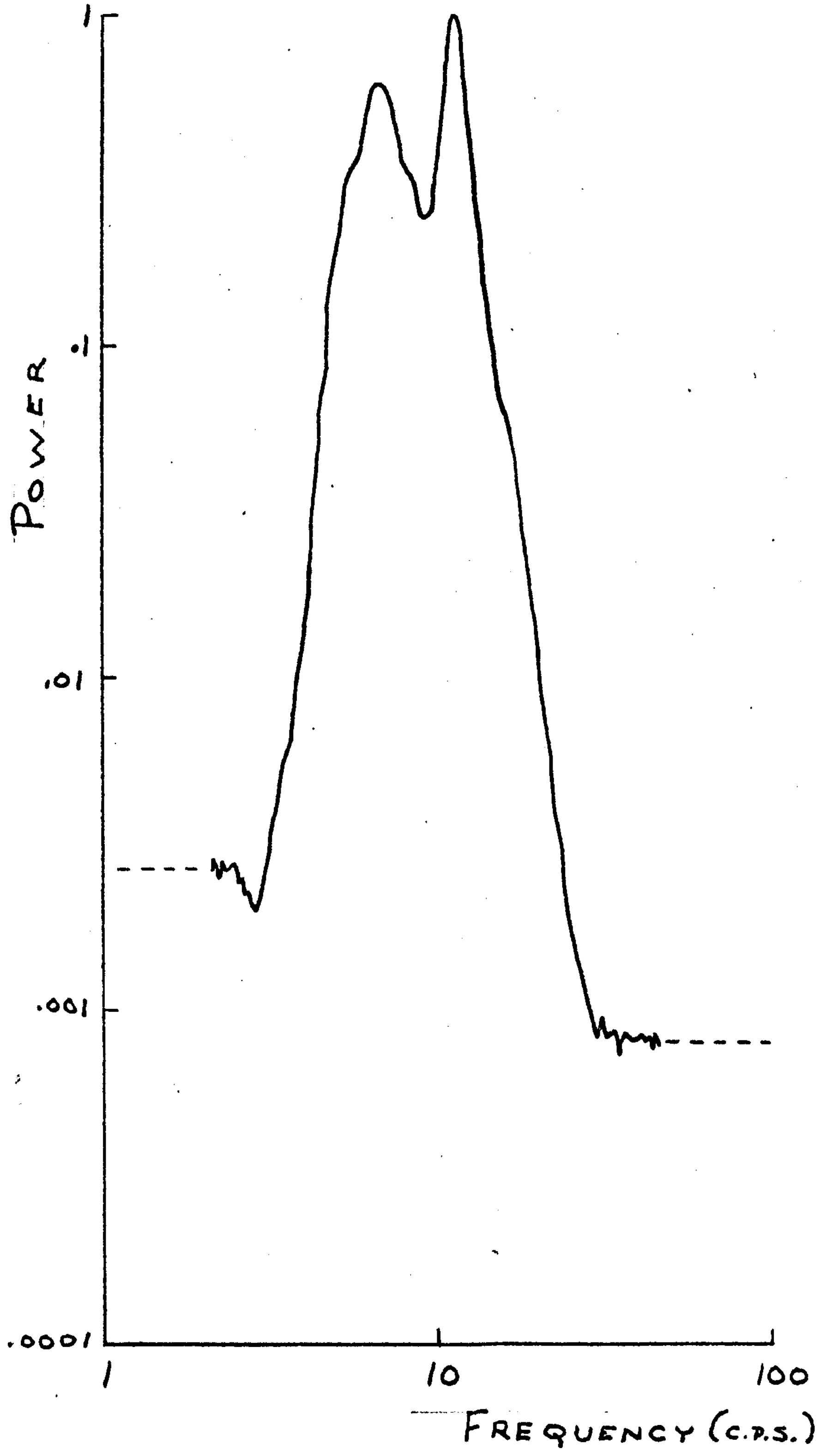

FIGURE 5 — TYPICAL POWER SPECTRUM (ACTUAL RECORD)

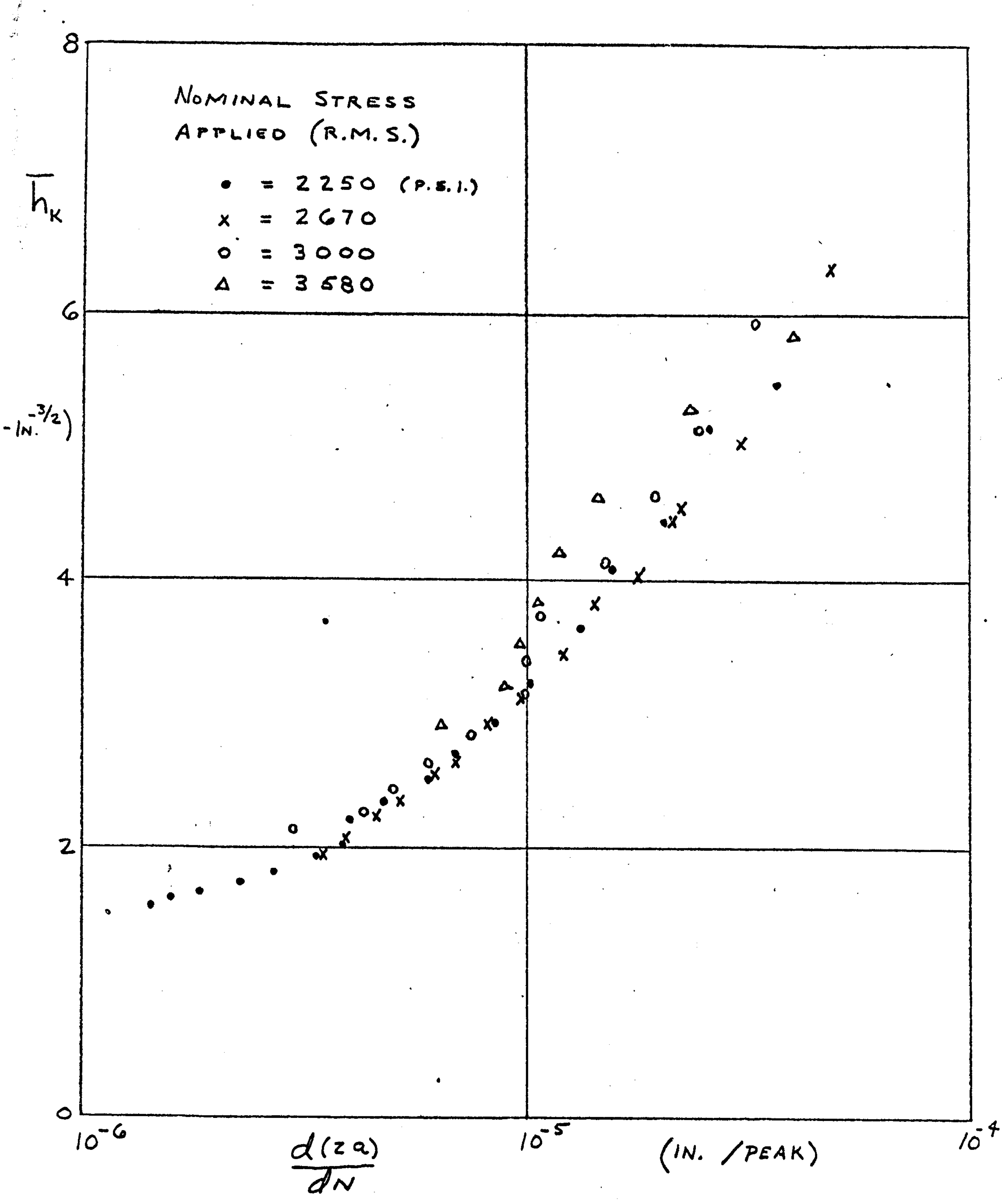

FIGURE 6 — CORRELATION OF 7075-T6

DATA FOR EQUIVELENT SPECTRA

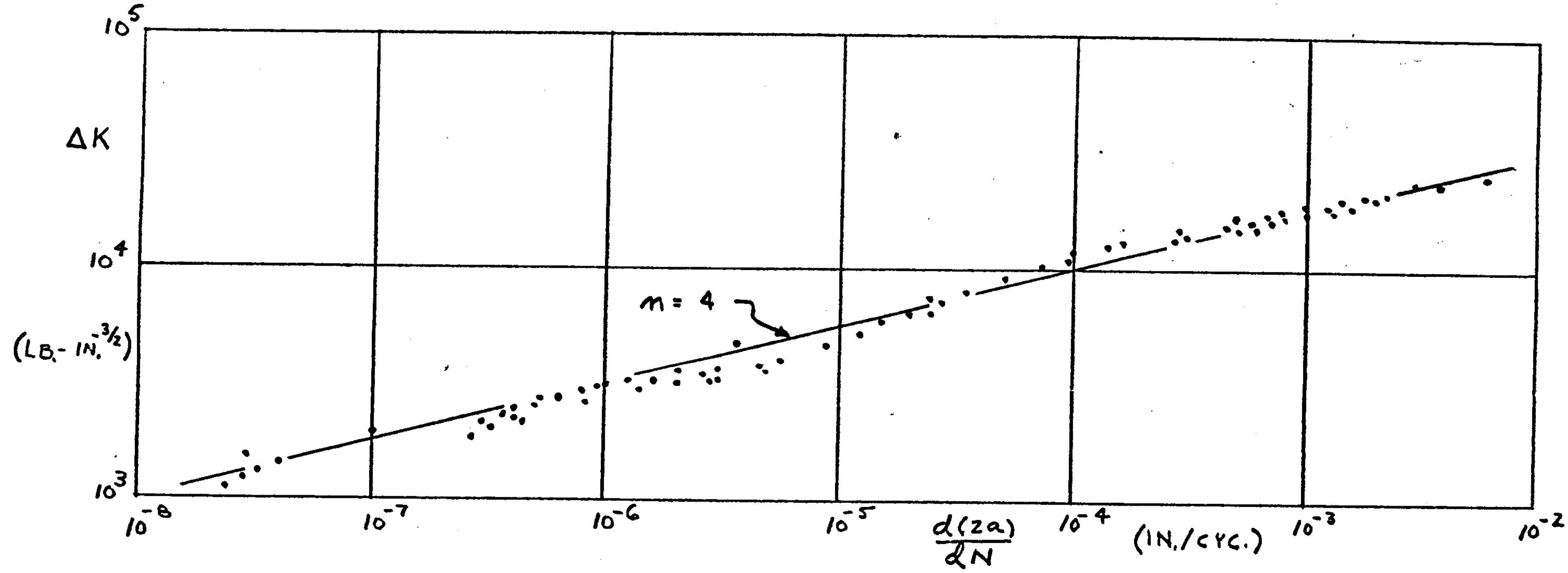

FIGURE 7 — 7075-T-6 DATA

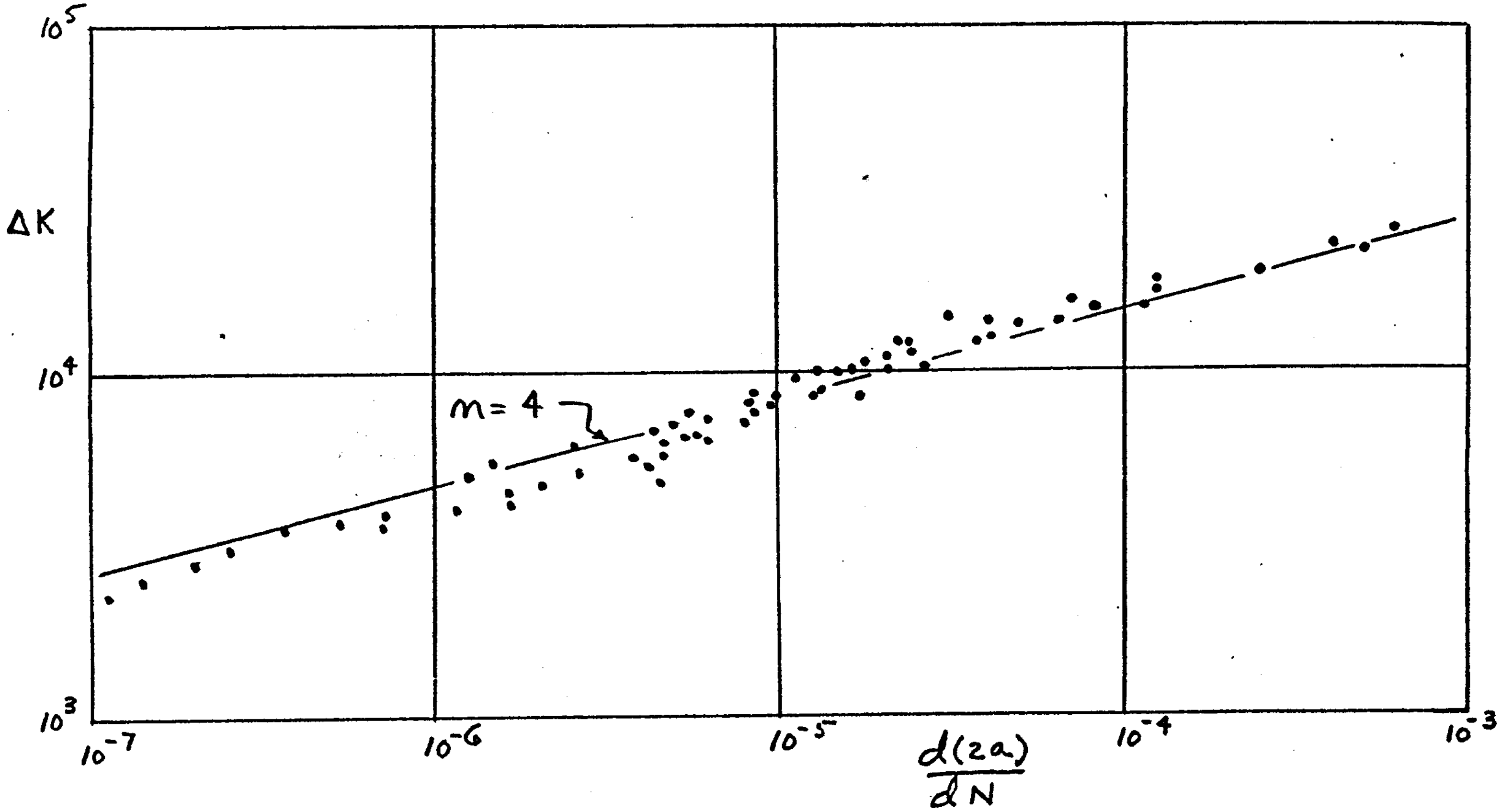

FIGURE 8 – 2024-T3 DATA

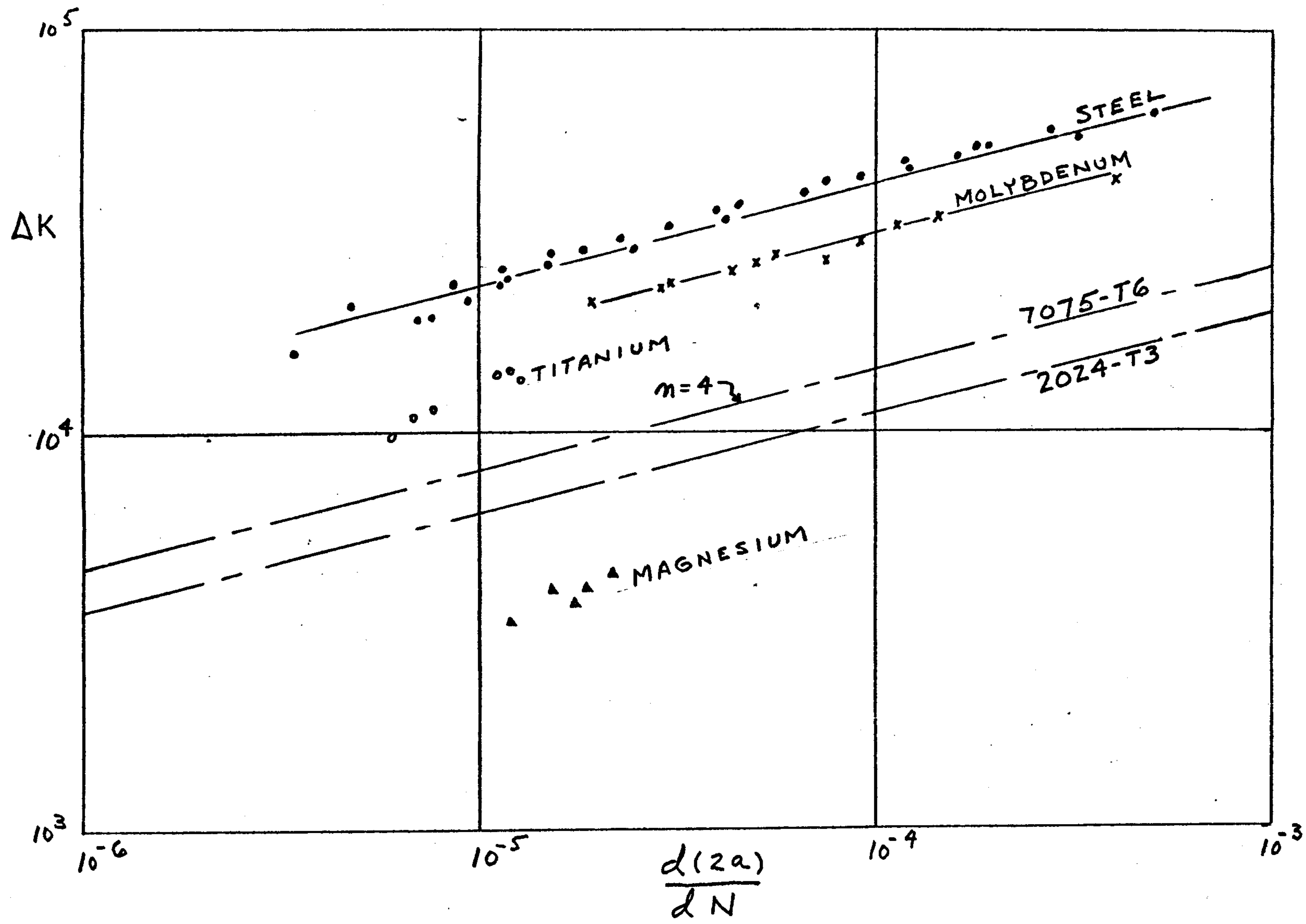

FIGURE 9 — DATA ON VARIOUS MATERIALS

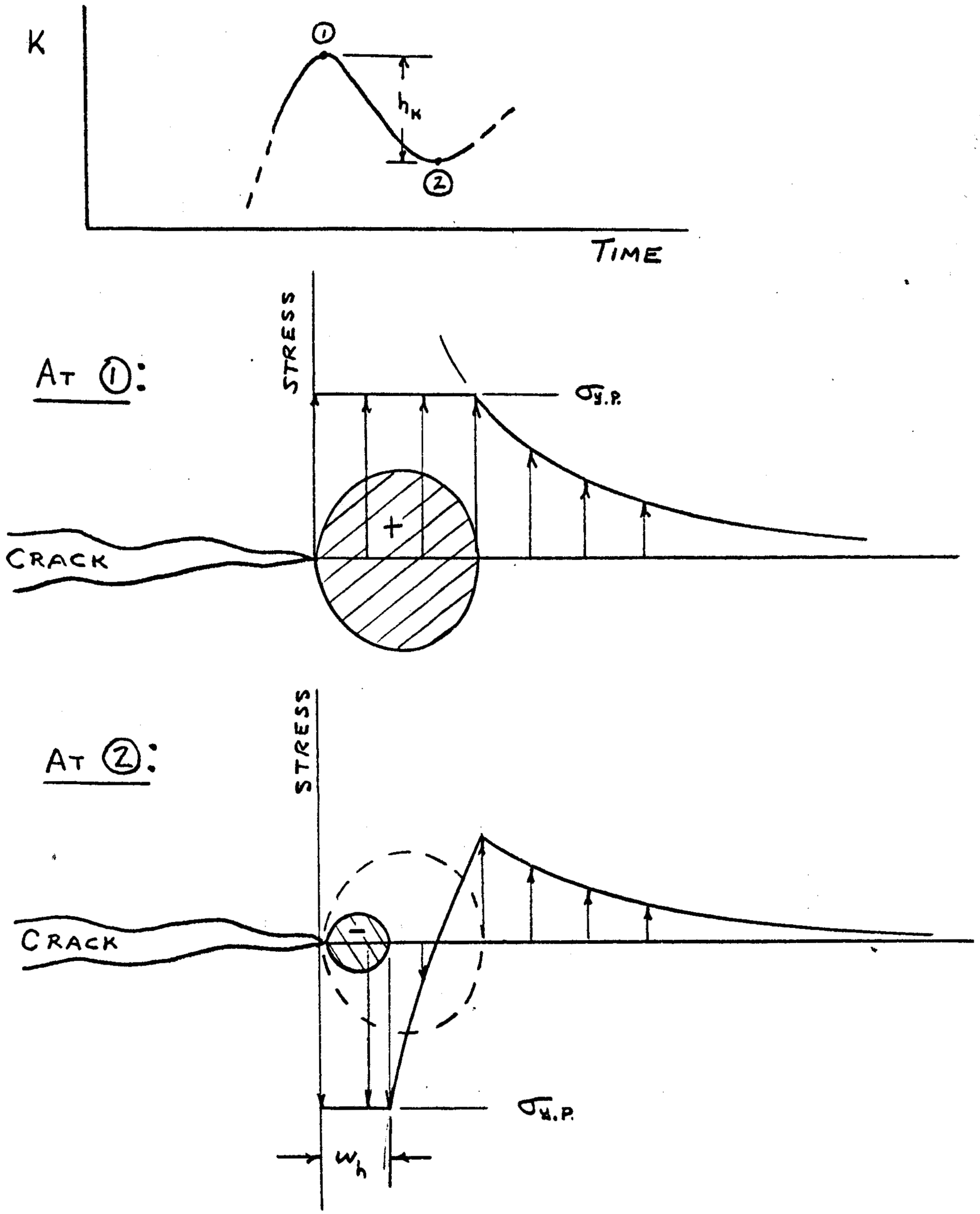

FIGURE 10 — SUBSEQUENT PLASTICITY

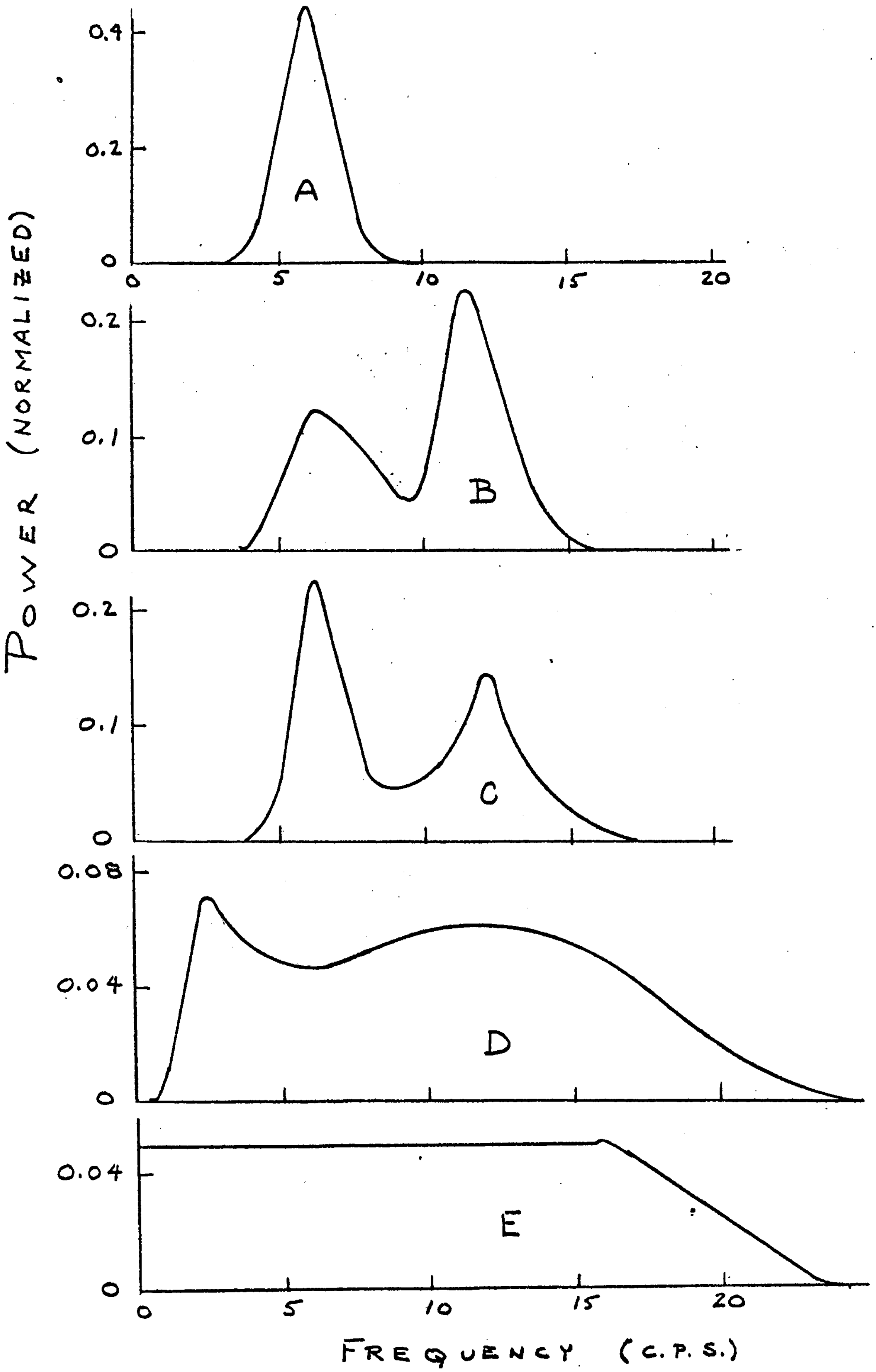

FIGURE 11 — POWER SPECTRA

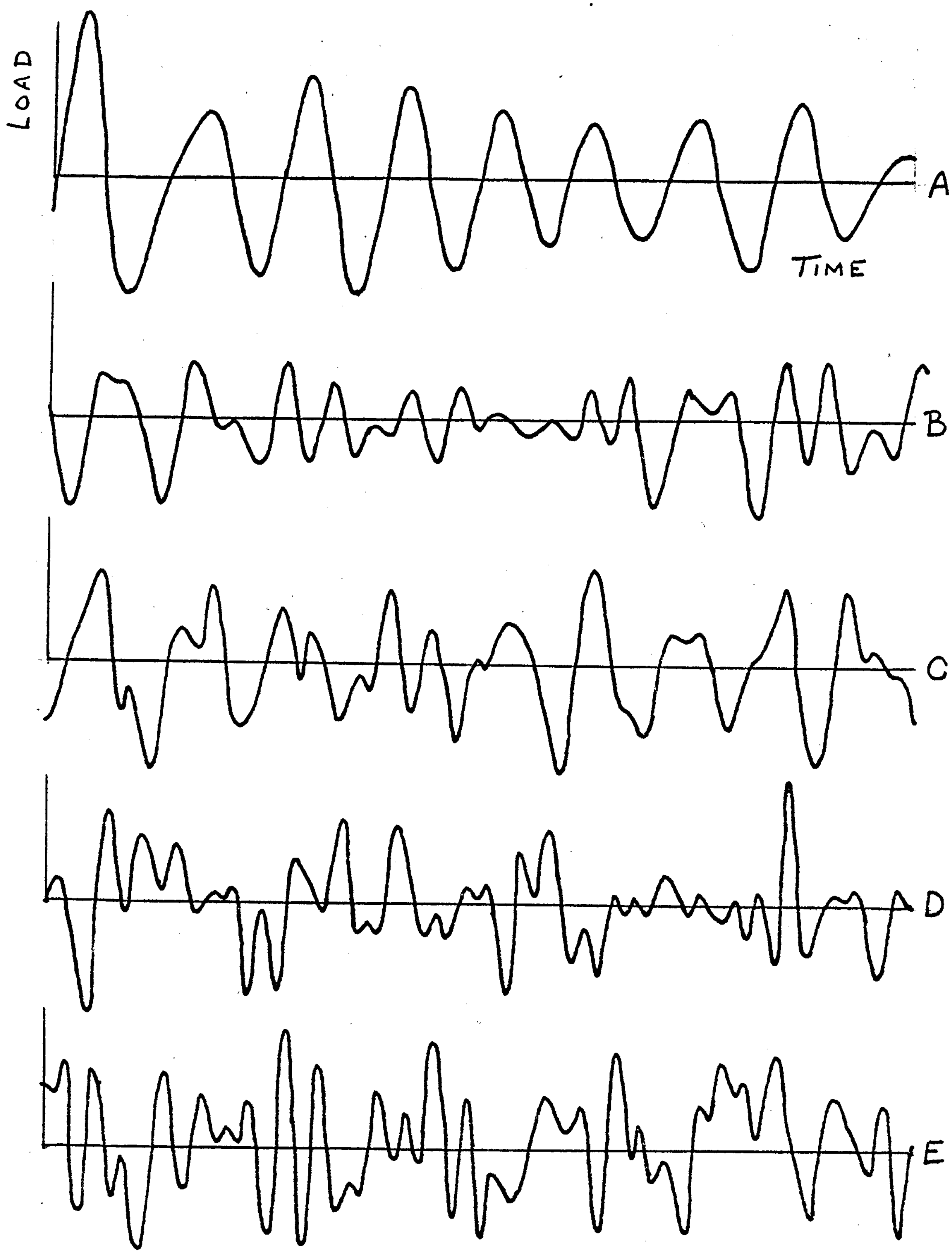

FIGURE 12 - LOAD-TIME HISTORIES

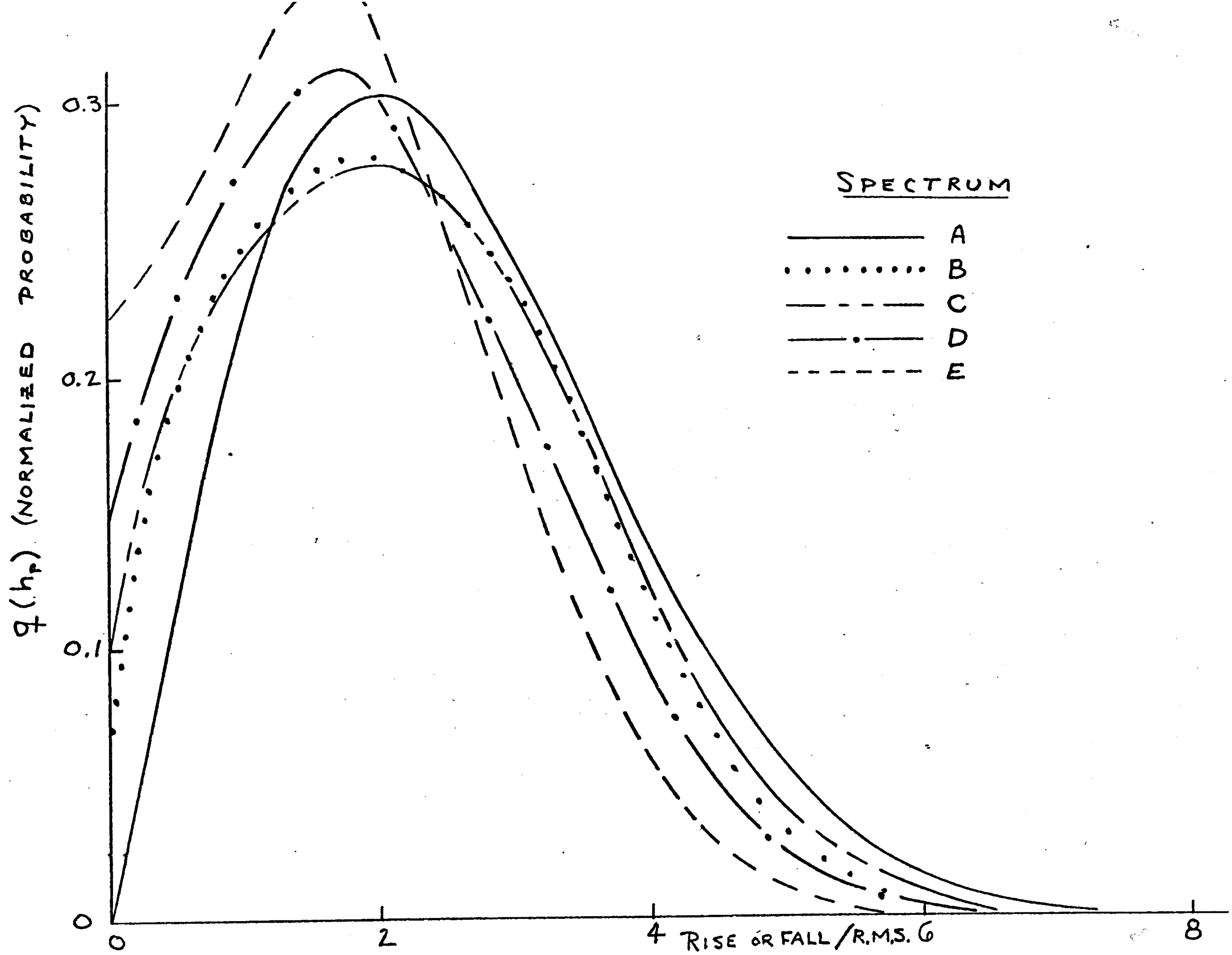

FIGURE 13 — RISE AND FALL DISTRIBUTION

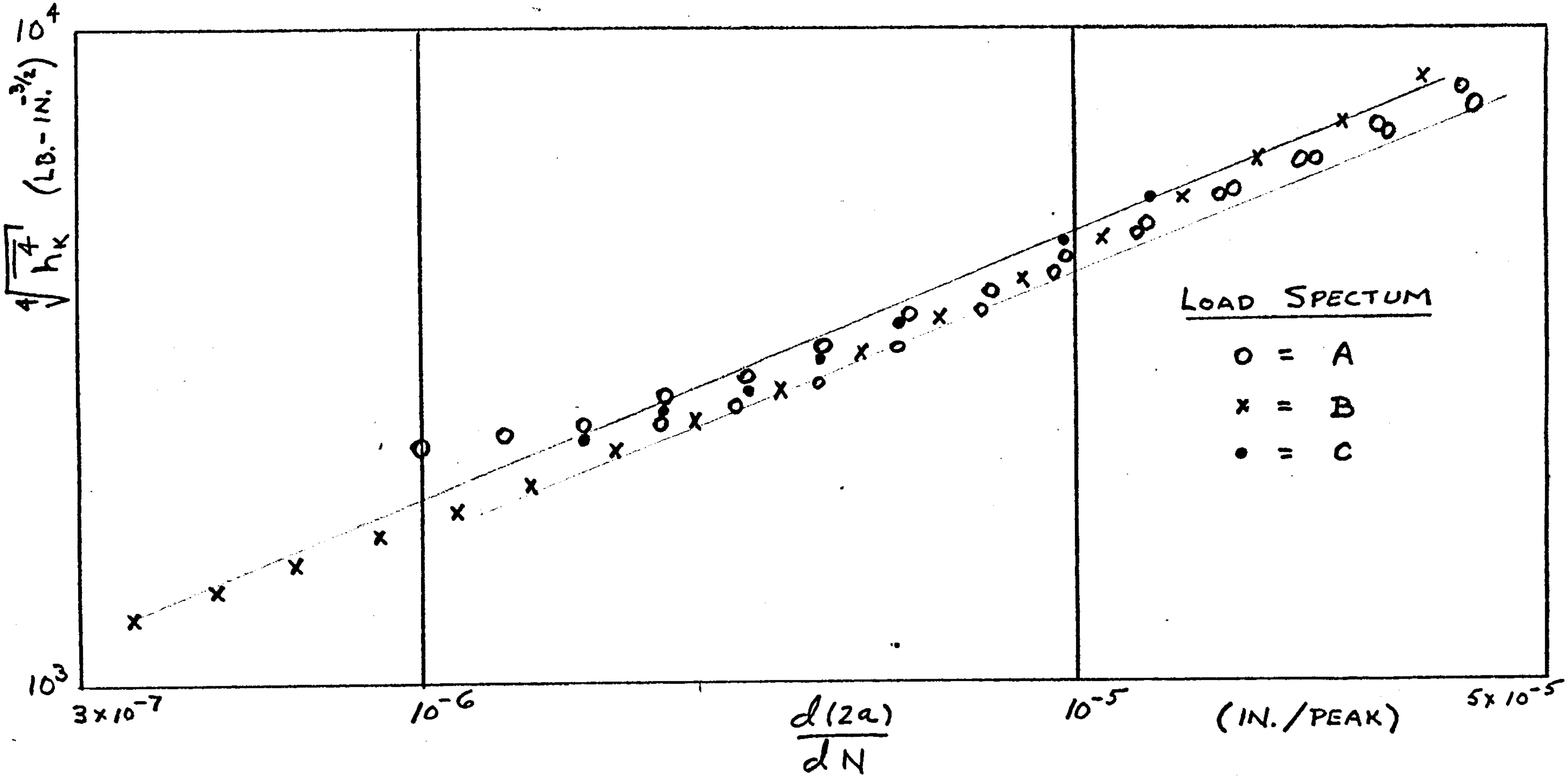

FIGURE 14 — GROWTH RATES UNDER VARIOUS RANDOM LOADS

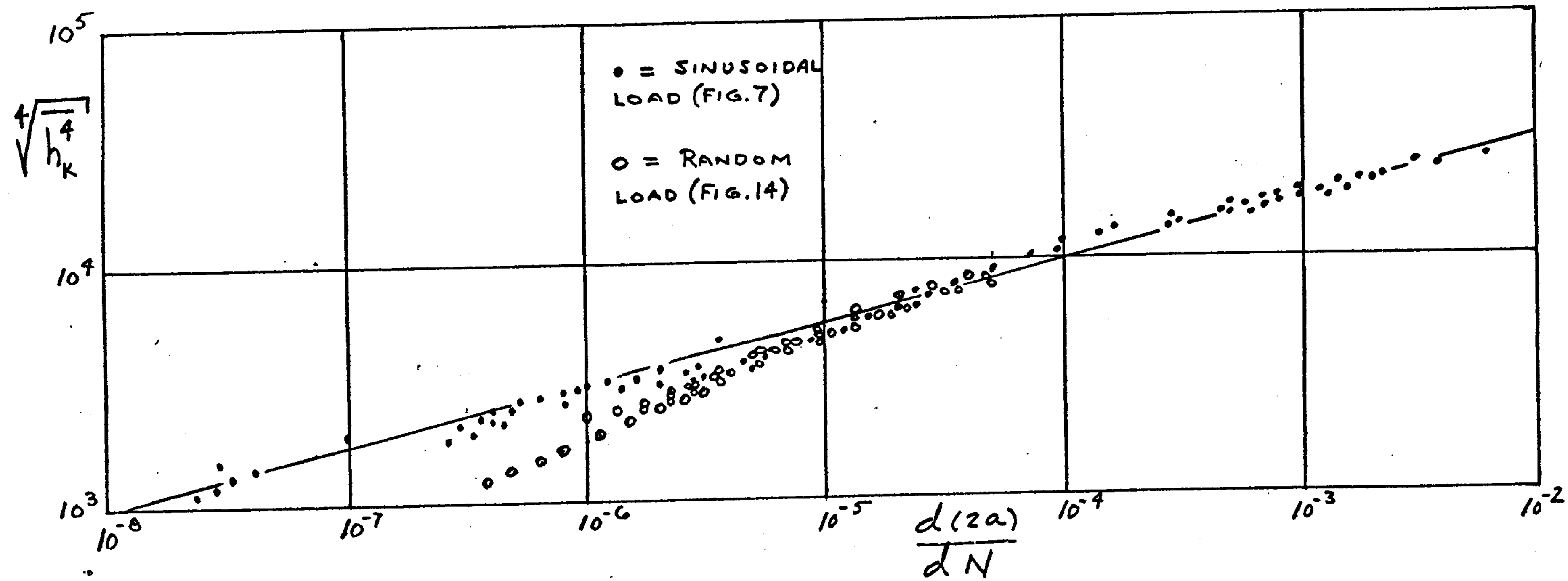

FIGURE 15 — 7075-T6 RANDOM AND SINUSOIDAL LOADING TESTS

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NOTATION

a	=	Half Crack Length.
C, C_i	=	Constants (depending on material, relative mean load, and frequency).
$\frac{d(2a)}{dN}$	=	Crack Growth per cycle or peak in load.
$f(a)$	=	A factor relating the load to the stress intensity factor; a function configuration including crack length.
F	=	Wedge forces applied to a crack surface.
$F^T(\omega)$	=	Frequency spectrum.
h_k, h_p	=	Rise(and/or fall) between peaks in stress-intensity-factor and load, respectively.
i	=	$\sqrt{-1}$
$K, \Delta K, K_{mean}$	=	The stress-intensity-factor, its range and its mean value.
M_r	=	The rth moment of a power spectrum.
N	=	Cycle number or number of peaks.
$P, \Delta P, P_{mean}$	=	Load, its range and its mean.
$q(\ )$	=	Probability density function of.
r, θ	=	Polar Coordinates measured from a crack tip.
t	=	Time.

$S_k(\omega), S_p(\omega)$	=	Power spectra of stress-intensity-factor and load, respectively.
W	=	Plastic zone width.
$\mathcal{W}$	=	Plastic work dissipated.
γ	=	Relative mean load.
σ	=	Uniform axial stress.
$\sigma_x, \sigma_y, \tau_{xy}$	=	Rectangular stress components.
$\sigma_{y.p.}$	=	Yield point stress.
ω	=	Circular frequency (2π C.P.S.).
$\overline{}$	=	(Bar) denotes an average.